## 6. Comparison with elastic analytical solution

As a first verification of the mesh convergence, numerical solutions in elasticity are made in 3D model to compare with the results obtained from the analytical and numerical solution in plane state of deformation of Guo et al [6]. These authors developed an analytical solution for the stress field around unlined twin tunnels with hydrostatic pressure in elastic medium. Fig. 12 illustrates the main parameters of the domain.

Figure 12: Parameters, domain and boundary conditions of the Guo et al. [6] analysis.

The results are shown at crown (point B) and sidewall (point A) of the tunnel considering 𝑅𝑡 = 4 m, 𝐸 = 500 MPa, 𝜈 = 0.23, 𝜎𝑣 = 𝜎ℎ = 2.2 MPa. The Fig. 13 shows the convergence profiles in the crown for various longitudinal tunnel axis distances 𝑑1 normalized by the longitudinal tunnel diameter 2𝑅𝑡 . It can be seen that the closer the longitudinal tunnels, the greater the convergence at the crown. The solution for a single tunnel was made using an axisymmetric model, but this result is also obtained using Corbetta’s analytical formulations [36].

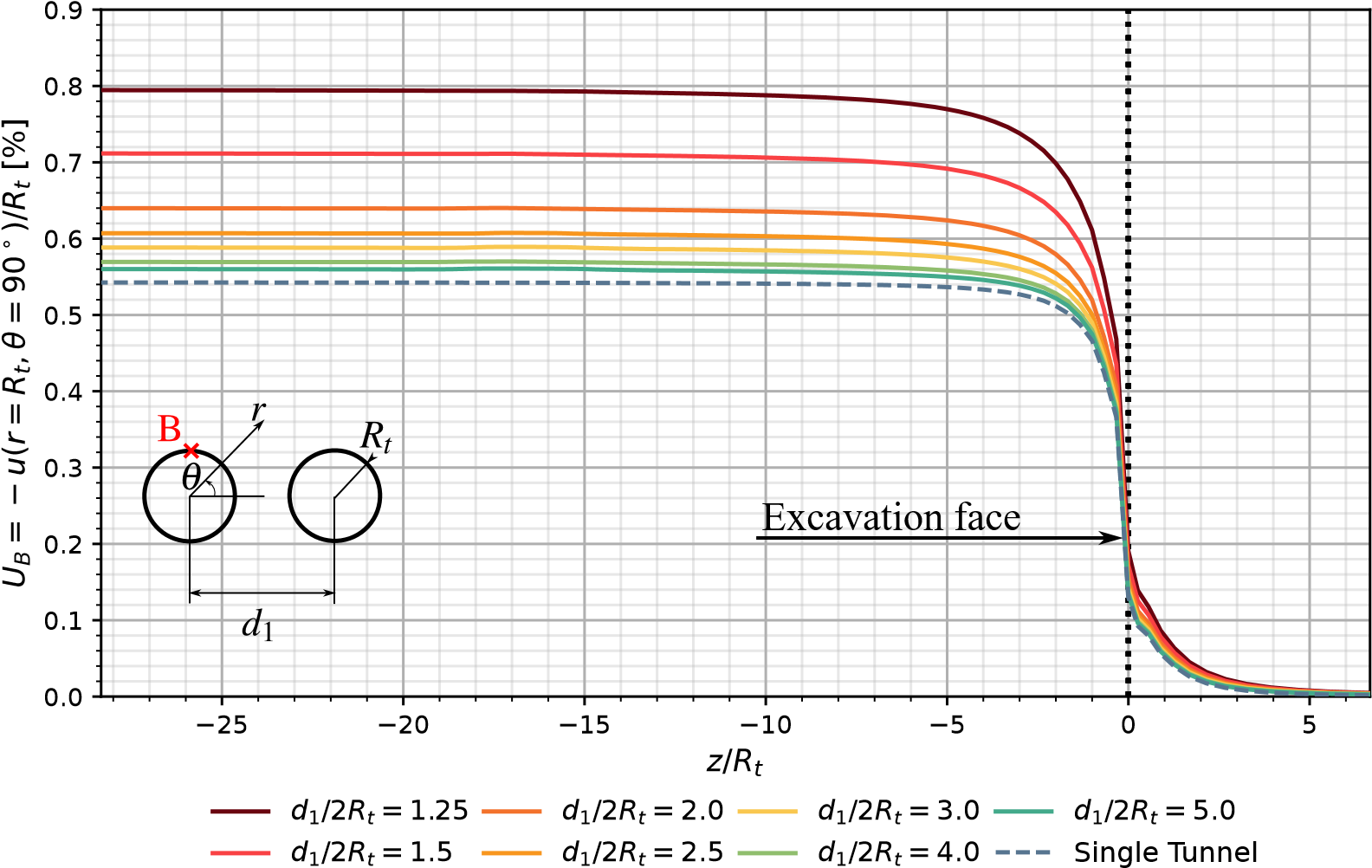
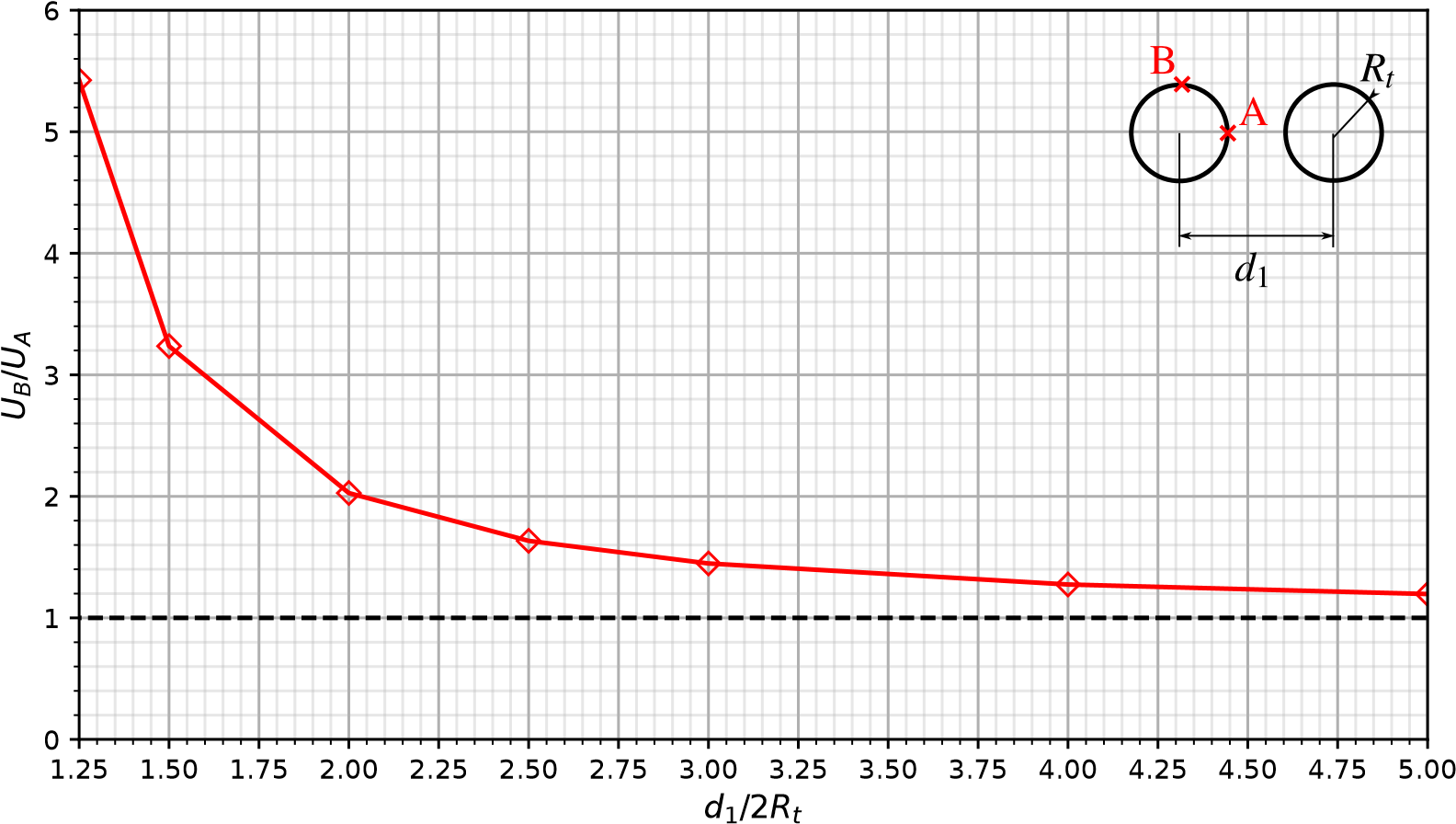
Figure 13: Convergence profiles in the crown (point B).

Fig. 14 shows the relationship between the convergences in the crown and sidewall, showing the ovalization effect

that occurs when the longitudinal tunnels get closer and closer together.



Present Analysis Single Tunnel

Figure 14: Relationship between convergence in the crown (point B) and sidewall (point A).

Fig. 15 shows the analytical and numerical solutions of Guo et al [6] (in blue and green color) together with the results obtained with the current 3D model (in red color). Ling’s solution [37], employed by these researchers to validate their analytical and numerical approaches, is also shown (in black color). Extra analyses with *𝑑*1∕2*𝑅𝑡* = 4 and 5 show that, as the longitudinal tunnels become increasingly spaced apart, the solution converges to that of a single tunnel (in dashed black color).

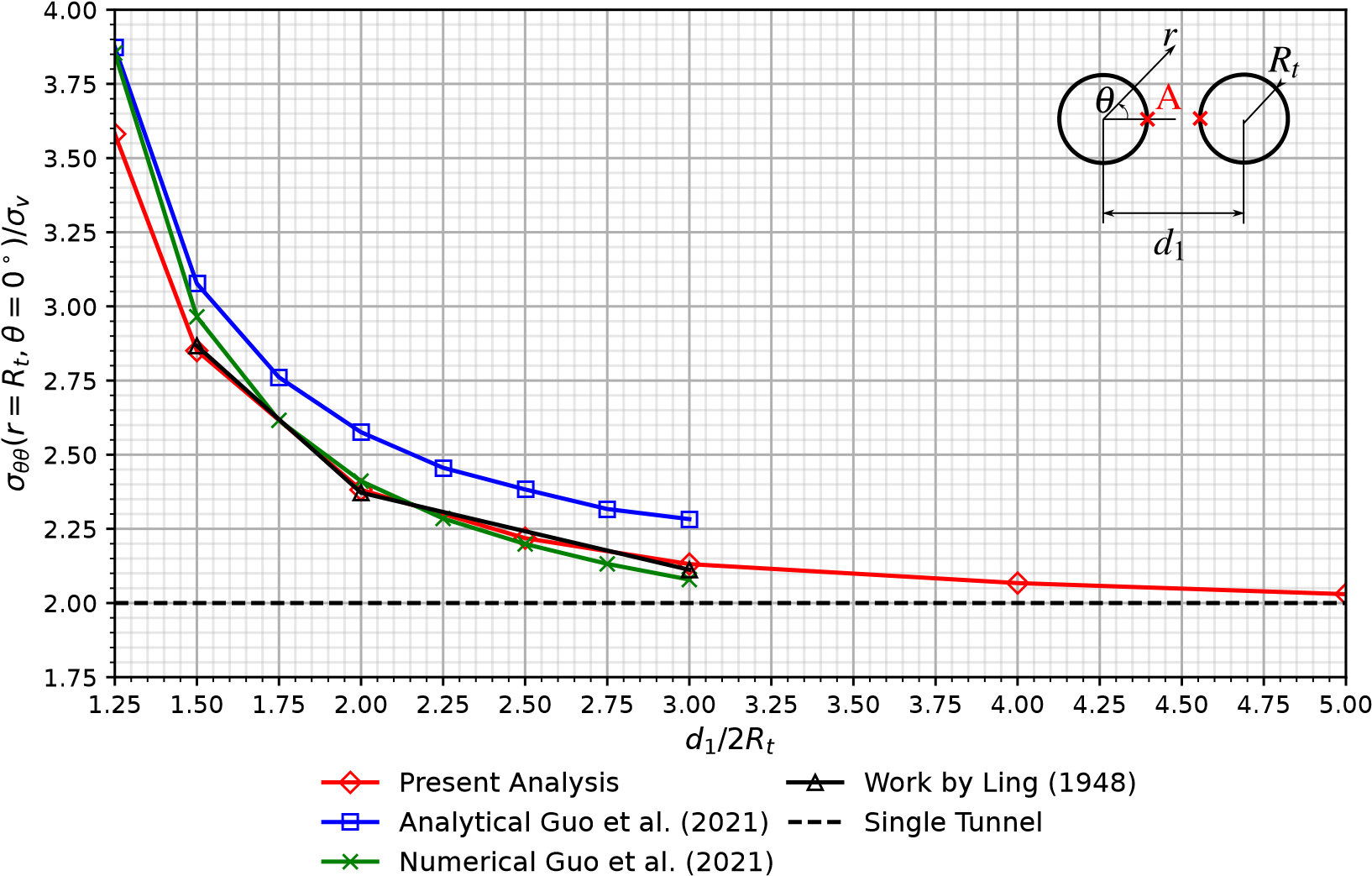
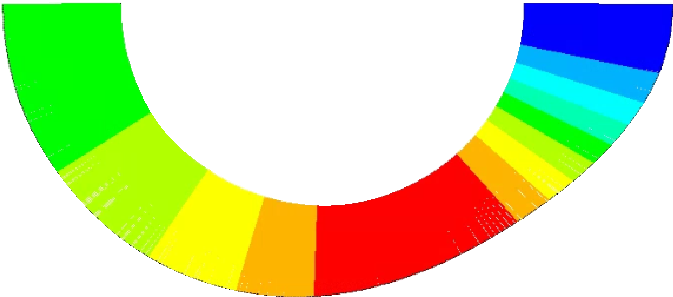
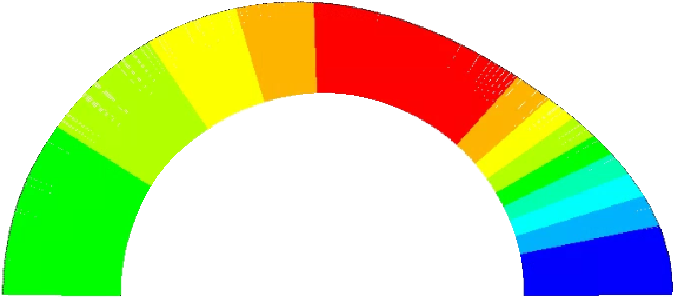


Figure 15: Tangential stress concentration factor in the sidewall (point A).

The Fig. 16 shows the comparison of the tangential stress distribution around the tunnel’s boundary considering

*𝑑*1∕2*𝑅𝑡* = 1*.*5.



-3.80

Current

D model

3

solution

Analytical Solution

Guo, et al. (2021)

-6.30

-5.99

-5.69

-5.38

-5.07

-4.76

-4.45

-4.15

-3.84

-3.53

-5.00

-4.97

-4.13

-3.80

-3.74

-6.72

Figure 16: Verification of numerical results of tangential stresses with the analytical solution in elasticity.

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